Multiplying Fractions with Fractions

The meaning of multiplying a fraction by a fraction is to take that fractional part of the fraction. When a fraction is multiplied by a fraction, **multiply the numerators and multiply the denominators**. It is best to cancel before multiplying. Canceling is a way to simplify fractions before multiplying.

NOTE

The number properties of whole numbers (Associative, Commutative, and Distributive) are still applicable with fractions.

Example 1

Find the product: $\frac{4}{9} \times \frac{2}{5} =$

Explanation

$$\frac{4}{9} \times \frac{2}{5} = \frac{4 \times 2}{9 \times 5} = \frac{8}{45}$$

Example 2

Find the product: $\frac{25}{28} \times \frac{8}{35} =$

Explanation

$$\frac{25}{28} \times \frac{8}{35} = \frac{\overset{5}{\cancel{25}}}{\underset{7}{\cancel{28}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{7}{\cancel{35}}} = \frac{10}{49}$$

Common mistakes

$$\frac{25}{28} \times \frac{8}{35} = \frac{25}{\underset{4}{\cancel{28}}} \times \frac{8}{\underset{5}{\cancel{35}}} = \frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}}} = 10$$ ✗

Explanation: Canceling is between a numerator and a denominator.

Next we are going to introduce a few examples that are little more challenging.

Example 3

Apply the necessary property to solve: $\frac{3}{5} \times \frac{6}{13} + \frac{3}{5} \times \frac{7}{13} =$

Explanation

Use the distributive property:

$$\frac{3}{5} \times \frac{6}{13} + \frac{3}{5} \times \frac{7}{13} = \frac{3}{5} \times (\frac{6}{13} + \frac{7}{13}) = \frac{3}{5}$$

Example 4

Apply the necessary property to solve: $\frac{16}{45} \times \frac{11}{18} - \frac{11}{45} \times \frac{7}{18} =$

Explanation

Switch the numerators in $\frac{16}{45} \times \frac{11}{18}$ or $\frac{11}{45} \times \frac{7}{18}$ so that the number property can be applied to simplify the calculation:

$$\frac{16}{45} \times \frac{11}{18} - \frac{11}{45} \times \frac{7}{18}$$
$$= \frac{11}{45} \times \frac{16}{18} - \frac{11}{45} \times \frac{7}{18}$$
$$= \frac{11}{45} \times (\frac{16}{18} - \frac{7}{18})$$
$$= \frac{11}{45} \times \frac{9}{18}$$
$$= \frac{11}{90}$$

or

$$\frac{16}{45} \times \frac{11}{18} - \frac{11}{45} \times \frac{7}{18}$$
$$= \frac{16}{45} \times \frac{11}{18} - \frac{7}{45} \times \frac{11}{18}$$
$$= \frac{11}{18} \times (\frac{16}{45} - \frac{7}{45})$$
$$= \frac{11}{18} \times \frac{9}{45}$$
$$= \frac{11}{90}$$

When a fraction is multiplied by a fraction, multiply the numerators and multiply the denominators.
Therefore the product remains the same if we switch the positions of numerators.

Example 5

Apply the necessary strategy to solve: $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \cdots + \frac{1}{99 \times 100} =$

Explanation

Every fraction can be transformed into the format of $\frac{1}{a \times (a + 1)}$. And then $\frac{1}{a \times (a + 1)} = \frac{1}{a} - \frac{1}{a + 1}$.

So $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \cdots + \frac{1}{99 \times 100} = (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + \cdots + (\frac{1}{99} - \frac{1}{100})$

Remove the parentheses and use the associative property.

$$(\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + \cdots + (\frac{1}{99} - \frac{1}{100})$$
$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{99} - \frac{1}{100}$$
$$= \frac{1}{2} - \frac{1}{100}$$
$$= \frac{49}{100}$$